MSSM Higgs boson production via gluon fusion: the large gluino mass limit

Margarete Mühlleitner,[a] **Heidi Rzehak**[b] **and Michael Spira**[c]

[a]*LAPTH, Université de Savoie,*
CNRS, BP110, F–74941 Annecy-le-Vieux Cedex, France

[b]*ITP, Universität Karlsruhe,*
D–76128 Karlsruhe, Germany

[c]*Paul Scherrer Institut,*
CH–5232 Villigen PSI, Switzerland

ABSTRACT: Scalar MSSM Higgs boson production via gluon fusion $gg \to h, H$ is mediated by heavy quark and squark loops. The higher order QCD corrections to these processes turn out to be large. The full supersymmetric QCD corrections have been calculated recently. In the limit of large SUSY masses a conceptual problem appears, i.e. the proper treatment of the large gluino mass limit. In this work we will describe the consistent decoupling of heavy gluino effects and derive the effective Lagrangian for decoupled gluinos.

Contents

1 Introduction

Higgs boson [1] searches belong to the primary motivations for present and future colliders. In the MSSM two isospin Higgs doublets are introduced for the masses of up- and down-type fermions [2]. After electroweak symmetry breaking five states are left as elementary Higgs particles, two CP-even neutral (scalar) particles h, H, one CP-odd neutral (pseudoscalar) particle A and two charged bosons $H^{\pm}$. At leading order (LO) the MSSM Higgs sector is fixed by two independent input parameters which are usually chosen as the pseudoscalar Higgs mass M_A and $\mathrm{tg}\beta = v_2/v_1$, the ratio of the two vacuum expectation values. Being lighter than the Z boson mass at LO, the one-loop and dominant two-loop corrections shift the upper bound of the light scalar Higgs mass to $M_h \lesssim 140\,\mathrm{GeV}$ [3]. The couplings of the various neutral Higgs bosons to fermions and gauge bosons, normalized to the SM Higgs couplings, are listed in table 1. The angle α denotes the mixing angle of the scalar Higgs bosons h, H. An important property of the bottom Yukawa couplings is their enhancement for large values of $\mathrm{tg}\beta$, while the top Yukawa couplings are suppressed for large $\mathrm{tg}\beta$ [4]. Thus, the top Yukawa couplings play a dominant role at small and moderate values of $\mathrm{tg}\beta$.

Usually the scalar superpartners $\tilde{f}_{L,R}$ of the left- and right-handed fermion components mix with each other. However, in this work we will neglect mixing effects. Thus the masses of the sfermion states $\tilde{f}_{L,R}$ are simply given by

$$M_{\tilde{f}_{L,R}}^2 = m_f^2 + M_{L,R}^2 \tag{1.1}$$

where m_f denotes the fermion mass and $M_{L/R}$ the soft supersymmetry-breaking sfermion mass parameters. The neutral scalar Higgs couplings $[\mathcal{H} = h, H]$ to non-mixing sfermions read [5]

$$
\begin{aligned}
g_{\tilde{f}_L \tilde{f}_L}^{\mathcal{H}} &= g_{\tilde{f}_R \tilde{f}_R}^{\mathcal{H}} = m_f^2 g_f^{\mathcal{H}} \\
g_{\tilde{f}_L \tilde{f}_R}^{\mathcal{H}} &= 0 \\
g_{\tilde{f}_i \tilde{f}_j}^{A} &= 0
\end{aligned}
\tag{1.2}
$$

ϕ		g_u^ϕ	g_d^ϕ	g_V^ϕ
SM	H	1	1	1
MSSM	h	$\cos\alpha/\sin\beta$	$-\sin\alpha/\cos\beta$	$\sin(\beta-\alpha)$
	H	$\sin\alpha/\sin\beta$	$\cos\alpha/\cos\beta$	$\cos(\beta-\alpha)$
	A	$1/\mathrm{tg}\beta$	$\mathrm{tg}\beta$	0

Table 1. Higgs couplings in the MSSM to fermions and gauge bosons $[V = W, Z]$ relative to the SM couplings.
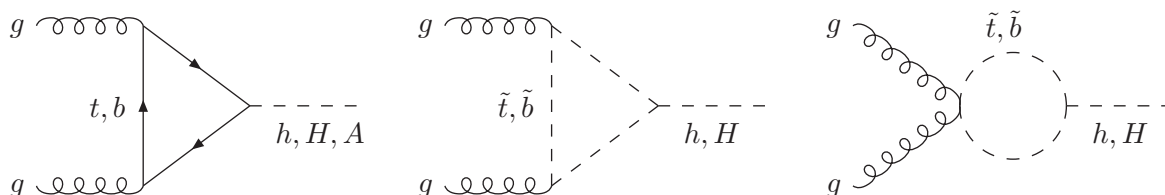

Figure 1. MSSM Higgs boson production via gluon fusion mediated by top- and bottom quark as well as stop and sbottom loops at leading order.

with the couplings $g_f^{\mathcal{H}}$ listed in table 1. D terms have been neglected in these expressions. It is important to note that supersymmetry relates the diagonal couplings to the corresponding fermion Yukawa coupling involving the fermion mass m_f.

At hadron colliders as the Tevatron and LHC neutral Higgs bosons are copiously produced by the gluon fusion processes $gg \to h/H/A$, which are mediated by top and bottom quark loops as well as stop and sbottom loops for the scalar Higgs bosons h, H in the MSSM (see figure 1) [6].

The pure QCD corrections to the (s)top and (s)bottom quark loops are known including the full Higgs and (s)quark mass dependences [7]. They increase the cross sections by up to about 100%. The limit of very heavy top quarks and squarks provides an approximation within $\sim 20-30\%$ for $\mathrm{tg}\beta \lesssim 5$ [8]. In this limit the next-to-leading order (NLO) QCD corrections have been calculated [9] and later the next-to-next-to-leading order (NNLO) QCD corrections [10]. The NNLO corrections lead to a further moderate increase of the cross section by $\sim 20-30\%$, so that the dominant part are the NLO contributions. An estimate of the next-to-next-to-next-to-leading order (NNNLO) effects has been obtained [11] indicating improved perturbative convergence. Moreover, the full SUSY-QCD corrections have been derived for heavy SUSY particle masses [12–15] and recently including the full mass dependence [16]. Ref. [12] addresses the limit of large gluino masses for the scalar Higgs couplings to gluons for degenerate squark masses, i.e. without mixing, as a special limit of their final result. The result develops a logarithmic singularity for large gluino masses which seems to contradict the decoupling of the gluino contributions according to the Appelquist-Carazzone theorem [17]. In the pseudoscalar Higgs case this logarithmic

divergence for large gluino masses does not appear [14]. This work describes the resolution of this problem and a consistent derivation of the effective Lagrangian after decoupling the gluino contributions.

The paper is organized as follows. In section 2 we derive the effective Lagrangian in the limit of heavy quark, squark and gluino masses, where the gluinos are much heavier than the quarks and squarks in addition. Section 3 summarizes and concludes.

2 Decoupling of the gluinos

For the derivation of the effective Lagrangian for the scalar Higgs couplings to gluons we will analyze the relation between the quark Yukawa coupling λ_Q and the Higgs coupling to squarks $\lambda_{\tilde{Q}}$ in the limit of large gluino masses in detail. To set up our notation we will define these couplings at leading order in the case of vanishing squark mixing as

$$
\begin{aligned}
\lambda_Q &= g_Q^{\mathcal{H}} \frac{m_Q}{v} \\
\lambda_{\tilde{Q}} &= 2 \frac{g_{\tilde{f}_L \tilde{f}_L}^{\mathcal{H}}}{v} = 2 \frac{g_{\tilde{f}_R \tilde{f}_R}^{\mathcal{H}}}{v} = 2 g_Q^{\mathcal{H}} \frac{m_Q^2}{v} = \kappa \lambda_Q^2 \\
\kappa &= 2 \frac{v}{g_Q^{\mathcal{H}}}
\end{aligned}
\tag{2.1}
$$

where $v = 1/\sqrt{\sqrt{2} G_F} \approx 246\,\text{GeV}$ denotes the vacuum expectation value of the Higgs sector which is related to the Fermi constant G_F. These couplings mediate the scalar Higgs decays into heavy quark pairs $\mathcal{H} \to Q\bar{Q}$ and into squark pairs $\mathcal{H} \to \tilde{Q}\bar{\tilde{Q}}$ (see figure 2a,b). In the following we will derive the modified relation between these couplings for scales *below* the gluino mass $M_{\tilde{g}}$. This will proceed along the following lines: *(i)* We will start with the unbroken relation between the running $\overline{MS}$ couplings of eq. (2.1) for scales *above* the gluino mass and the corresponding renormalization group equations. *(ii)* If the scales decrease *below* the gluino mass the gluino decouples from the renormalization group equations. This decoupling leads to modified renormalization group equations which differ for the two couplings $\lambda_{\tilde{Q}}$ and $\kappa \lambda_Q^2$. This implies that the two couplings deviate for scales *below* the gluino mass, while they are identical for scales *above* the gluino mass. *(iii)* The proper matching at the gluino mass scale yields a finite threshold contribution for the evolution from the gluino mass to smaller scales, while the logarithmic structure of the matching relation is given by the solution of the renormalization group equations *below* the gluino mass scale. We will determine these ingredients in detail in this letter. The matching relations can be obtained from the gluino contributions for heavy gluino masses in the limit of vanishing external momentum transfers [18] as we will discuss in the next sections in detail.

2.1 $\phi \to Q\bar{Q}$

The gluino contribution to the Higgs vertex with heavy quark pairs is depicted in figure 2c. We use dimensional regularization in $n = 4 - 2\epsilon$ dimensions for the evaluation of the one-loop contributions. Since all virtual particles are massive, there are no infrared divergences. Although dimensional regularization requires the introduction of anomalous
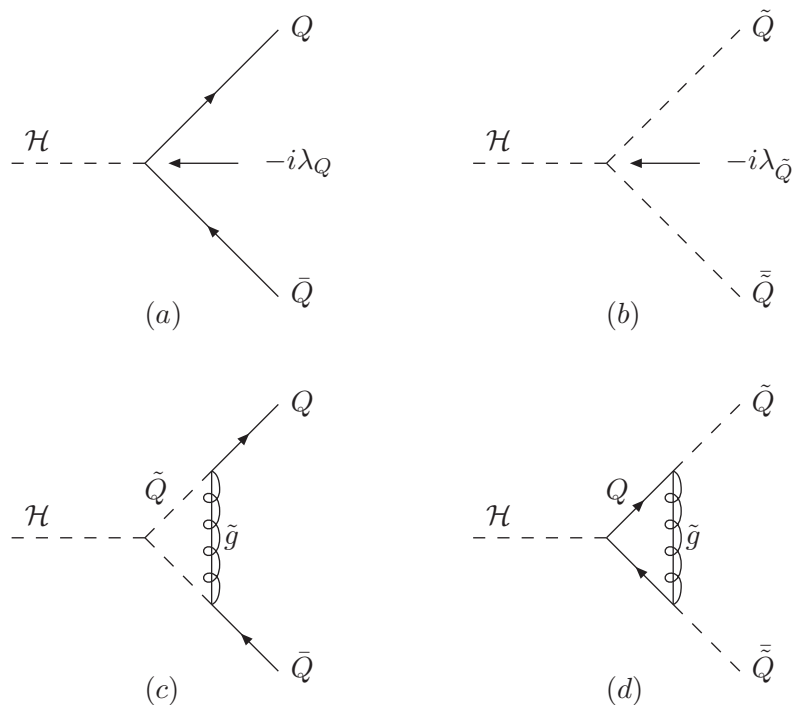
Figure 2. Scalar MSSM Higgs boson couplings to heavy quarks Q and squarks $\tilde{Q}$: (a) $\mathcal{H}Q\bar{Q}$ coupling at LO, (b) $\mathcal{H}\tilde{Q}\bar{\tilde{Q}}$ coupling at LO, (c) gluino contribution to the $\mathcal{H}Q\bar{Q}$ coupling at NLO and (d) gluino contribution to the $\mathcal{H}\tilde{Q}\bar{\tilde{Q}}$ coupling at NLO. [$\mathcal{H} = h, H$].
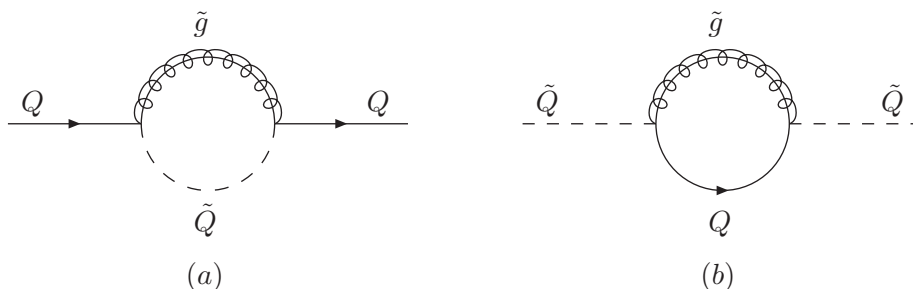

Figure 3. Gluino contributions to the self-energies of (a) quarks Q and (b) squarks $\tilde{Q}$ at NLO.

counter terms in general to restore the supersymmetric relations between corresponding couplings and masses, these gluino contributions are free of these terms. The result of the vertex contribution in the heavy gluino mass limit vanishes,

$$Z_1 - 1 \to 0 \tag{2.2}$$

The full correction to the bare Yukawa coupling requires the addition of the wave function renormalization constant Z_2 which can be derived from the derivative of the corresponding self-energy diagram shown in figure 3a. In the limit of large gluino masses the gluino

contribution is found to be $[C_F = 4/3]$

$$Z_2 - 1 \rightarrow C_F \frac{\alpha_s}{\pi} \Gamma(1+\epsilon) \left(\frac{4\pi\mu^2}{M_{\tilde{g}}^2} \right)^\epsilon \left\{ -\frac{1}{4\epsilon} - \frac{3}{8} \right\} \tag{2.3}$$

The gluino contributions to the bare quark Yukawa coupling can now be derived as

$$\frac{\Delta\lambda_Q}{\lambda_Q} = Z_1 Z_2 - 1 \rightarrow C_F \frac{\alpha_s}{\pi} \Gamma(1+\epsilon) \left(\frac{4\pi\mu^2}{M_{\tilde{g}}^2} \right)^\epsilon \left\{ -\frac{1}{4\epsilon} - \frac{3}{8} \right\} \tag{2.4}$$

This result can be used to construct the renormalization group equation for the quark Yukawa coupling without the gluino contribution. The full renormalization group equation for the running $\overline{MS}$ coupling including gluon and gluino contributions is at leading order given by [19]

$$\mu_R^2 \frac{\partial \bar{\lambda}_Q(\mu_R)}{\partial \mu_R^2} = -\frac{C_F}{2} \frac{\alpha_s(\mu_R)}{\pi} \bar{\lambda}_Q(\mu_R) \tag{2.5}$$

It describes the scale dependence of the running $\overline{MS}$ coupling for scales larger than the quark, squark and gluino masses. However, if the gluino mass is large compared to the chosen renormalization scale, the gluino has to be decoupled from the renormalization group equation. This can be performed consistently by a momentum subtraction of the gluino contribution for vanishing momentum transfer while treating the quark and gluon contributions in the usual $\overline{MS}$ scheme [18]. It relates the momentum-subtracted Yukawa coupling $\bar{\lambda}_{Q,MO}$ to the full $\overline{MS}$ coupling $\bar{\lambda}_Q$ in the following way

$$\bar{\lambda}_{Q,MO}(\mu_R) = \bar{\lambda}_Q(\mu_R) \left\{ 1 + C_F \frac{\alpha_s}{\pi} \left(\frac{1}{4} \log \frac{M_{\tilde{g}}^2}{\mu_R^2} - \frac{3}{8} \right) \right\} \tag{2.6}$$

Differentiating this relation with respect to μ_R^2 yields the renormalization group evolution of the momentum-subtracted coupling corresponding to the low-energy result *without* the gluino,

$$\mu_R^2 \frac{\partial \bar{\lambda}_{Q,MO}(\mu_R)}{\partial \mu_R^2} = -\frac{3}{4} C_F \frac{\alpha_s(\mu_R)}{\pi} \bar{\lambda}_{Q,MO}(\mu_R) \tag{2.7}$$

Since the matching of the effective theory *below* the gluino mass scale to the full MSSM will be performed at $\mu_R = M_{\tilde{g}}$, eq. (2.6) determines the required threshold contribution, too,

$$\bar{\lambda}_{Q,MO}(M_{\tilde{g}}) = \bar{\lambda}_Q(M_{\tilde{g}}) \left\{ 1 - \frac{3}{8} C_F \frac{\alpha_s(M_{\tilde{g}})}{\pi} \right\} \tag{2.8}$$

In ref. [12] the quark Yukawa coupling has been renormalized by introducing the quark pole mass m_Q. In the effective theory *below* the gluino mass scale the running $\overline{MS}$ coupling is related to the quark pole mass as [20]

$$g_Q^\phi \frac{m_Q}{v} = \bar{\lambda}_{Q,MO}(m_Q) \left\{ 1 + C_F \frac{\alpha_s(m_Q)}{\pi} \right\} \tag{2.9}$$

2.2 $\phi \to \tilde{Q}\bar{\tilde{Q}}$

The analogous calculation has to be repeated for the Higgs coupling to squarks $\lambda_{\tilde{Q}}$. The corresponding gluino contribution is shown in figure 2d. In this case no anomalous counter terms are needed, too. The result of the vertex correction in the heavy gluino mass limit is given by

$$\tilde{Z}_1 - 1 \to C_F \frac{\alpha_s}{\pi} \Gamma(1+\epsilon) \left(\frac{4\pi\mu^2}{M_{\tilde{g}}^2}\right)^\epsilon \left\{\frac{1}{\epsilon} + 1\right\} \tag{2.10}$$

Here we concentrate only on the diagonal terms, i.e. the $\mathcal{H}\tilde{q}_L\bar{\tilde{q}}_L$ and $\mathcal{H}\tilde{q}_R\bar{\tilde{q}}_R$ couplings, in the no-mixing case since these will be treated by the renormalization of the Yukawa coupling. In the general case including squark mixing effects the additional diagonal and non-diagonal contributions will be absorbed by the renormalized trilinear coupling A_Q. The gluino contribution to the squark wave function renormalization constant Z_2 for large gluino masses can be derived from the derivative of the corresponding self-energy diagram as depicted in figure 3b,

$$\tilde{Z}_2 - 1 \to C_F \frac{\alpha_s}{\pi} \Gamma(1+\epsilon) \left(\frac{4\pi\mu^2}{M_{\tilde{g}}^2}\right)^\epsilon \left\{-\frac{1}{2\epsilon} - \frac{1}{4}\right\} \tag{2.11}$$

In the case of squark mixing the additional non-diagonal contributions arising from the self-energy diagram of figure 3b will be absorbed by the renormalized mixing angle $\theta_{\tilde{q}}$. As for the quark Yukawa coupling the gluino contribution to the Higgs coupling to squarks can now be determined,

$$\frac{\Delta\lambda_{\tilde{Q}}}{\lambda_{\tilde{Q}}} = \tilde{Z}_1\tilde{Z}_2 - 1 \to C_F \frac{\alpha_s}{\pi} \Gamma(1+\epsilon) \left(\frac{4\pi\mu^2}{M_{\tilde{g}}^2}\right)^\epsilon \left\{\frac{1}{2\epsilon} + \frac{3}{4}\right\} \tag{2.12}$$

For the running $\overline{MS}$ Higgs coupling to squarks the full renormalization group equation including gluon, squark and gluino contributions can be expressed as [19]

$$\mu_R^2 \frac{\partial\bar{\lambda}_{\tilde{Q}}(\mu_R)}{\partial\mu_R^2} = -C_F \frac{\alpha_s(\mu_R)}{\pi} \bar{\lambda}_{\tilde{Q}}(\mu_R) \tag{2.13}$$

Note that by virtue of supersymmetry the beta function is twice as large as the corresponding one for the quark Yukawa coupling in eq. (2.5). Eq. (2.13) describes the evolution for scales above the quark, squark and gluino masses. The momentum-subtracted coupling $\bar{\lambda}_{\tilde{Q},MO}$ of the effective theory without gluinos is now related to the full $\overline{MS}$ coupling $\bar{\lambda}_{\tilde{Q}}$ by

$$\bar{\lambda}_{\tilde{Q},MO}(\mu_R) = \bar{\lambda}_{\tilde{Q}}(\mu_R)\left\{1 - C_F\frac{\alpha_s}{\pi}\left(\frac{1}{2}\log\frac{M_{\tilde{g}}^2}{\mu_R^2} - \frac{3}{4}\right)\right\} \tag{2.14}$$

where the additional contributions of quarks, gluons and squarks to $\bar{\lambda}_{Q,MO}$ are still $\overline{MS}$ subtracted. The evolution of the momentum-subtracted coupling is fixed by the renormalization group equation, which can be derived by differentiating eq. (2.14) with respect to μ_R^2,

$$\mu_R^2 \frac{\partial\bar{\lambda}_{\tilde{Q},MO}(\mu_R)}{\partial\mu_R^2} = -\frac{C_F}{2}\frac{\alpha_s(\mu_R)}{\pi}\bar{\lambda}_{\tilde{Q},MO}(\mu_R) \tag{2.15}$$

This renormalization group equation differs from the corresponding renormalization group equation of the squared momentum-subtracted Yukawa coupling $\bar\lambda_{Q,MO}$ of eq. (2.7). Thus the supersymmetric relation between these two couplings is violated for scales *below* the gluino mass where the gluino contribution has to be deleted from the corresponding beta functions. This difference is expected, since for heavy decoupled gluinos the residual contributing particle spectrum is not supersymmetric any more. Moreover, for the matching scale $\mu_R = M_{\tilde g}$, eq. (2.14) determines the threshold correction for the evolution *below* the gluino mass scale,

$$\bar\lambda_{\tilde Q,MO}(M_{\tilde g}) = \bar\lambda_{\tilde Q}(M_{\tilde g})\left\{1 + \frac{3}{4}C_F\frac{\alpha_s(M_{\tilde g})}{\pi}\right\} \tag{2.16}$$

Due to the decoupling of the gluinos the soft supersymmetry breaking induces a hard supersymmetry breaking at low energy scales [21] as can be inferred from the different threshold corrections and the different renormalization group equations *below* the gluino mass scale.

2.3 Decoupling of gluinos

Now we are in the position to derive the effective low-energy scalar Higgs coupling to gluons. For the consistent decoupling of heavy gluinos their contribution has to be treated in the momentum-subtraction scheme as described before. The relation between the momentum-subtracted quark Yukawa coupling and the scalar Higgs coupling to squarks can be determined as follows. In the supersymmetric theory, i.e. for scales *above* the gluino mass, the supersymmetric relation holds for the running $\overline{MS}$ couplings,

$$\kappa\bar\lambda_Q^2(\mu_R) = \bar\lambda_{\tilde Q}(\mu_R) \tag{2.17}$$

Using eqs. (2.6), (2.14) this leads to a non-supersymmetric relation between the momentum-subtracted couplings,

$$\kappa\bar\lambda_{Q,MO}^2(m_Q) = \bar\lambda_{\tilde Q,MO}(m_Q)\left\{1 + C_F\frac{\alpha_s}{\pi}\left(\log\frac{M_{\tilde g}^2}{m_Q^2} - \frac{3}{2}\right)\right\} \tag{2.18}$$

In this equation we have set the renormalization scale equal to the quark mass, since at this scale the momentum-subtracted Yukawa coupling $\bar\lambda_{Q,MO}$ is related to the quark pole mass. Using eq. (2.9) for the relation between the quark pole mass and the $\overline{MS}$ Yukawa coupling of the effective theory below the gluino mass scale we arrive at the relation

$$2g_Q^{\mathcal H}\frac{m_Q^2}{v} = \bar\lambda_{\tilde Q,MO}(m_Q)\left\{1 + C_F\frac{\alpha_s}{\pi}\left(\log\frac{M_{\tilde g}^2}{m_Q^2} + \frac{1}{2}\right)\right\} \tag{2.19}$$

The proper scale choice for the effective Higgs coupling to squarks, however, is the squark mass. This choice is relevant for an additional large gap between the quark and squark masses. Using the renormalization group equation of eq. (2.15) we end up with the final relation

$$2g_Q^{\mathcal H}\frac{m_Q^2}{v} = \bar\lambda_{\tilde Q,MO}(m_{\tilde Q})\left\{1 + C_F\frac{\alpha_s}{\pi}\left(\log\frac{M_{\tilde g}^2}{m_{\tilde Q}^2} + \frac{3}{2}\log\frac{m_{\tilde Q}^2}{m_Q^2} + \frac{1}{2}\right)\right\} \tag{2.20}$$

The radiative corrections to the relation between the effective couplings after decoupling the gluinos modify the final result of ref. [12]. This modification can be discussed in terms of the effective Lagrangian in the limit of heavy squarks and quarks,

$$\mathcal{L}_{eff} = \frac{\alpha_s}{12\pi} G^{a\mu\nu} G^a_{\mu\nu} \frac{\mathcal{H}}{v} \left\{ \sum_Q g_Q^{\mathcal{H}} \left[1 + \frac{11}{4} \frac{\alpha_s}{\pi} \right] + \sum_{\tilde{Q}} \frac{g_{\tilde{Q}}^{\mathcal{H}}}{4} \left[1 + C_{SQCD} \frac{\alpha_s}{\pi} \right] + \mathcal{O}(\alpha_s^2) \right\} \quad (2.21)$$

where $g_{\tilde{Q}}^{\mathcal{H}} = v \bar{\lambda}_{\tilde{Q},MO}(m_{\tilde{Q}})/m_{\tilde{Q}}^2$. In ref. [12] the leading term of the supersymmetric coefficient for large gluino masses has been derived for equal squark masses $m_{\tilde{Q}} = M_{\tilde{Q}_L} = M_{\tilde{Q}_R}$ as

$$C_{SQCD}^{HS} = \frac{11}{2} - \frac{4}{3} \log \frac{M_{\tilde{g}}^2}{m_{\tilde{Q}}^2} - 2 \log \frac{m_{\tilde{Q}}^2}{m_Q^2} \quad (2.22)$$

where the mismatch between the couplings of eq. (2.20) for scales below the gluino mass has not been taken into account, i.e. keeping the relation $g_{\tilde{Q}}^{\mathcal{H}} = 2 g_Q^{\mathcal{H}} m_Q^2 / m_{\tilde{Q}}^2$ after renormalization as in the supersymmetric limit. Expressing $g_{\tilde{Q}}^{\mathcal{H}}$ in terms of $\bar{\lambda}_{\tilde{Q},MO}(m_{\tilde{Q}})$ instead, this mismatch leads to the additional contribution

$$\Delta C_{SQCD} = \frac{4}{3} \log \frac{M_{\tilde{g}}^2}{m_{\tilde{Q}}^2} + 2 \log \frac{m_{\tilde{Q}}^2}{m_Q^2} + \frac{2}{3} \quad (2.23)$$

Adding both contributions $C_{SQCD} = C_{SQCD}^{HS} + \Delta C_{SQCD}$ determines the supersymmetric contribution to the effective Lagrangian,

$$C_{SQCD} = \frac{37}{6} \quad (2.24)$$

The resulting effective Lagrangian is well-defined in the limit of large gluino masses and thus fulfills the constraints of the Appelquist-Carazzone decoupling theorem [17].

The resulting effective Higgs coupling $\bar{\lambda}_{\tilde{Q},MO}(m_{\tilde{Q}})$ can be determined by solving the renormalization group equations eqs. (2.7), (2.15) which are valid for scales *below* the gluino mass. Taking into account the proper matching to the fully supersymmetric $\overline{MS}$ coupling at the scale $\mu_R = M_{\tilde{g}}$ according to eqs. (2.8), (2.16) we arrive at the expression

$$\bar{\lambda}_{\tilde{Q},MO}(m_{\tilde{Q}}) = 2 g_Q^{\mathcal{H}} \frac{m_Q^2}{v} \frac{1 + \frac{3}{2} C_F \frac{\alpha_s(M_{\tilde{g}})}{\pi}}{1 + 2 C_F \frac{\alpha_s(m_Q)}{\pi}} \left(\frac{\alpha_s(M_{\tilde{g}})}{\alpha_s(m_{\tilde{Q}})} \right)^{\frac{C_F}{\beta_0}} \left(\frac{\alpha_s(m_{\tilde{Q}})}{\alpha_s(m_Q)} \right)^{\frac{3 C_F}{2 \beta_0}} \quad (2.25)$$

where $\beta_0 = (33 - 2 N_F - N_{\tilde{F}})/12$ denotes the leading order beta function of the strong coupling α_s. N_F is the number of contributing quarks and $N_{\tilde{F}}$ the number of contributing squark flavours. Decoupling only the gluino from the supersymmetric spectrum $[N_F = N_{\tilde{F}} = 6]$ its value is given by $\beta_0 = 5/4$. The expression (2.25) can be used to evaluate the effective coupling $\bar{\lambda}_{\tilde{Q},MO}(m_{\tilde{Q}})$ from the quark pole mass to leading logarithmic accuracy. This expression resums the leading logarithms of the gluino mass and provides the consistent matching of the low-energy Lagrangian to the full MSSM. In this way the value of the gluino mass is still measurable as the scale at which the two couplings $\bar{\lambda}_{\tilde{Q},MO}$

and $\kappa\bar{\lambda}^2_{Q,MO}$ merge after taking into account the corresponding threshold corrections. Note that the pure perturbative expansion of eq. (2.25) reproduces eq. (2.20) up to $\mathcal{O}(\alpha_s)$. Finally we would like to emphasize that eqs. (2.20), (2.25) do also hold in the general mixing case for the part of the Higgs couplings to squarks which is related to the quark Yukawa coupling, if the individual squark masses are chosen as the renormalization scales.

Since pseudoscalar Higgs bosons only couple to different squark states, i.e. to $\tilde{Q}_L\bar{\tilde{Q}}_R$ and vice versa, there are no squark loops at LO so that the pseudoscalar coupling to squarks contributes at NLO for the first time. This explains, why the logarithmic singularity for large gluino masses does not appear in the pseudoscalar case at NLO [14].

3   Conclusions

In this work we have described the consistent decoupling of heavy gluino mass effects from the effective Lagrangian for the scalar Higgs couplings to gluons within the MSSM. We have shown that a careful extraction of the gluino contributions in the large gluino mass limit induces a modification of the supersymmetric relations between the Higgs couplings to quarks and squarks. This modification involves non-trivial logarithmic gluino mass contributions to the effective couplings. They exactly cancel the left over gluino mass logarithms of the previous work of ref. [12] which did not take into account the mismatch between the Higgs couplings to quarks and squarks at scales below the gluino mass. This work ensures that the gluino contributions decouple explicitly for large gluino masses in accordance with the Appelquist-Carazzone theorem [17]. The gluino mass remains in the effective low-energy theory as the matching scale to the full MSSM. Analogous methods have to be applied to all processes if decoupling properties are analyzed.

Acknowledgments

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